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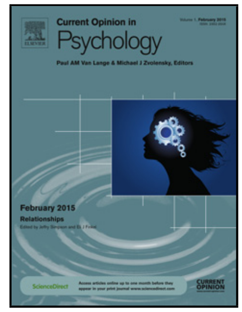

Title: Accounting for attention in sequential sampling models of decision making

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Accounting for attention in sequential sampling models of decision making

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Abstract
When making decisions, people tend to shift their attention back and forth between stimuli, choosing options that they look at more overall and immediately prior to their responses. These relationships, and others, are well-described by sequential sampling models that assume that evidence for a given alternative is collected over time in proportion to its subjective value, amplified by attention. Furthermore, findings from a number of studies support a causal effect of attention on choice. This research is mostly focused on two-alternative forced choice, though some work has confirmed these relationships in multi-attribute and multi-alternative choice. Finally, we discuss recent interest in understanding what drives attention during the choice process, with findings suggesting that attention is drawn to noisier stimuli and more salient stimuli in two-alternative choice, as well as higher-value options in multi-alternative choice.

Over the past few years, decision scientists have been working to understand the purpose of eye-movements during the choice process, using a combination of choice experiments and computational modeling. This research has identified relationships between gaze and choice, both over the course of the whole decision and at the time of choice. In other words, people tend to choose options that they've looked at first, are focused on at the time of choice, and have looked at more overall. These are the basic qualitative phenomena we seek to understand. There are however additional quantitative nuances and exceptions that are important, and help to refine our understanding of the choice process.

Here, we describe the core empirical relationships between gaze and choice, provide an overview of the models that have been used to explain these relationships, and discuss issues related to causality and domains of applicability.

Attention and choice

Coffee or tea? Apple or orange? Beer or wine? Chicken or fish? We know that such decisions are typically not instantaneous, nor are they perfectly predictable. These two phenomena are not independent. There is a consistent negative correlation between response times (RT) and the likelihood of making consistent choices. That is, if someone quickly chooses coffee over tea, but slowly chooses beer over wine, next time they would likely choose coffee again but maybe switch to wine [1]. The fact that decisions are probabilistic and take time suggests that there are idiosyncrasies from one decision to the next, even between the same options, that can influence the choice outcome. One key factor that might drive this variability is attention.

With that idea in mind, Krajbich et al. [2] ran an experiment in which participants made incentivized, two-alternative forced choices (2AFC) between familiar snack foods, while being eye-tracked. Separately, the authors also collected each participant's subjective-value ratings for each food. Participants' choices were not completely consistent with their ratings and accounting for their gaze data improved choice predictions. In particular, individual dwell times as well as the overall relative dwell time for an option were both predictive of choice (Fig. 1a). Also, at the time of choice, participants were more likely to choose an option if they were looking at it than if they weren't (Fig. 1b). Notice that this does not necessarily imply that participants were always more likely to choose the last-seen option. In fact, participants were less likely to choose the last-seen option when it had a much lower value than the other alternative. Similar patterns have been replicated in many subsequent experiments [3–7].

These gaze patterns rule out simple stories such as "people internally make a decision, look to that option, and then indicate their choice" or "gaze simply reflects value". Instead it appears as though value and attention interact to drive the decision process. To better understand how that might occur, we turn to modeling.

Sequential Sampling Models

Sequential sampling models (SSM) assume that during a decision, people evaluate their options, continuously "sampling" noisy information (or "evidence") about each's desirability. This information may come from the stimuli or from internal representations of the options. Sampling continues until the relative evidence for one option reaches a predetermined threshold. These models are a staple in cognitive psychology [8], have seen much support in neuroscience [9], and are increasingly being used for value/preference-based decisions [10]. They capture the speed-accuracy tradeoff, as well as the correlation between decision difficulty and RT.

Traditionally, it has been assumed that the average rate at which the relative evidence accumulates, i.e. the drift rate, is constant within a decision. Krajbich et al. [2] proposed instead that drift rate might change with gaze; the attentional drift diffusion model (aDDM), assumes that an option receives more evidence when gazed at, in line with recent neural data [11]. Decision field Theory (DFT) employs a very similar idea, where attention fluctuates between option attributes, determining the weights on those attributes in the overall drift rate [12–14]. The aDDM is able to accurately capture the relationship between dwell time and final-fixation on choice, as well as many other patterns in the data linking choice, RT, and gaze (Fig. 1c-d).

The aDDM (and analogously DFT) assumes that attention determines the weights on the evidence being gathered from the options. In other words, gaze has an amplifying effect on the attended option. An alternative possibility is that gaze merely adds evidence, providing a fixed advantage for the attended option [6,7]). A recent paper systematically compared these two models using six datasets [15]. The two models provide surprisingly similar fits to most aspects of the data, but they do differ in a couple important ways. Unlike the additive model, the aDDM correctly predicts that a decision between two high-value options will take less time and be more influenced by gaze than a decision between two low-value options (holding value difference constant; Fig. 2). Notably, these effects were less evident in the learning tasks with a small number of stimuli, consistent with the idea that participants may be using another strategy to solve those tasks. The aDDM account also correctly predicts that with aversive options, gaze should amplify the negative evidence, leading to a lower choice probability [16] (but see [6,15]).

Other work has investigated more complex SSMs incorporating attention [17,18]. This work indicates that additional model features such as mutual inhibition between evidence accumulators, evidence leakage, and a primacy effect, can improve model fits. However, in the Ashby paper, fits of the aDDM show nearly complete discounting of the unattended options, suggesting potential issues with their data or fitting methods, and the Colas paper does not explicitly consider gaze data. Other recent SSM work, not including attention, has also suggested that collapsing thresholds and/or increasing urgency to decide, may be important model features as they prevent difficult, decisions from taking too much time [19–21]. Empirically, it remains an open issue which combination of model features is the best, or if that depends on the specific decision task [22].

Causality

An important question that this research raises is the issue of causality. Research in other tasks indicates that value captures attention [23], so people often assume that this is what drives the relationship between gaze and choice in 2AFC. There is, however, substantial evidence in the opposite direction.

Before getting into details, it is worth noting that the aDDM itself is agnostic about causality. It is merely a mathematical mapping from value and gaze to choice and RT. It captures the fact that during certain stretches of time an option consistently receives more evidence than it does during other times. That being said, there is reason to believe that gaze does have a causal effect on choice, and not vice versa.

To establish the causal relationship of attention on choice, researchers have used various exogenous manipulations, including exposure time [16,24,25] (but see [26]), visual salience [27,28], the timing of decision prompts [5,29] (but see [30]), the location of consistently better items [31], and spatial cueing [32]. All these manipulations lead to corresponding choice biases. It is worth noting that the size of these effects varies and is typically less than what one would predict from the correlational data. The weakest effects tend to arise in the direct gaze manipulations, perhaps reflecting the fact that gaze and attention are not necessarily the same thing. Participants could continue thinking about Option A, even when forced to shift their gaze to Option B.

On the other hand, evidence for a causal effect of value on attention is rare. In their original study, Krajbich et al. [2] found that first fixations were equally likely to go to the higher or lower value options, and that the duration of a given dwell was uncorrelated with the value of the gazed-at option (see also [33]). The gaze-cascade effect, the phenomenon where attention leading up to the choice is biased towards the chosen option, was thought to be evidence of an effect of preference on attention [24] but is readily explained by the aDDM with random attention [34].

This is an important issue. A key assumption of SSMs is that the decision-maker does not know which option is better. Even as the decision evolves and one option begins to emerge as the favorite, it is still optimal to continue sampling information randomly, rather than favoring information from the leading option [35]. To put it another way, the goal is to separate the two options, so information about the trailing option is just as useful as information about the leading option. While this imposes a time cost, it yields an accuracy benefit.

There are some interesting cases where random fixations/dwells-times have not been observed. While these irregularities do not invalidate the aDDM/SSM per se, they do suggest that an alternative decision strategy may be at work. The cases where researchers have observed correlations between value and fixations/dwell-times are tasks where there are a small number of learned stimuli [3,6]. In these cases, participants may approach their decisions in a different way, using heuristics or planning ahead of time [4,3].

Multi-option/attribute choice

While much of the work on SSMs has focused on 2AFC, models such as DFT and the multiattribute linear ballistic accumulator (MLBA) [36] were designed to handle multiple options with multiple attributes, and there are multi-option extensions of the DDM [37]. Yet, only a few papers have considered gaze data in multi-attribute or multi-option models [38–40,37,27,41], and fewer still have explicitly incorporated gaze data into an SSM framework [27,37,38,41]

A basic challenge for this modeling is understanding the fixation process. With only two single-attribute options the problem is simple (but not easy); one must only account for when the participant switches to the other option. Some attempts have been made to understand this process, based on the idea of reducing uncertainty [42].

With multiple attributes/options, the problem becomes more complex. We require a model for what people switch to, one that accounts for changes over time. Early on, gaze is driven by spatial location and visual salience; people tend to gravitate to the center, or to the top/left if there is nothing in the center, and to brighter options [37,27,43]. Later on, gaze appears to be more influenced by value, as participants rule out certain options and focus on the leading options [26,44]. Unlike with 2AFC, it is not optimal to allocate equal attention to all of the options [45].

One way to sidestep the problem of modeling the fixation process is to go back to modeling a single drift rate per trial, but accounting for the total time spent on each stimulus. While this strategy ignores the dynamics of the gaze process, it does provide an elegant way to fit the model to 2AFC and multi-option data [6,41]

Multi-attribute, multi-option choice is a potentially even more interesting problem, as attention to different attributes may likely depend on the importance [46,33,47,48,26], ease of processing [36,49,50], and/or variability [51] of those attributes. This is still a relatively underexplored, but important area of research [52,44].

Conclusions

Here we have highlighted evidence for a relationship between attention and decision making, as captured by SSMs that exhibit increased evidence accumulation rates for attended stimuli. There is substantial evidence for a causal, amplifying effect of attention on choice, both in binary and multi-option cases. There are however many important questions left to answer. What factors affect individual differences in attentional discounting [41,4]? How do these phenomena play out in actual stores? What are the neural mechanisms underlying these effects [53,11,25]? We look forward to learning the answers to these questions and more in the years ahead.

Declarations of interest: none

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Reference annotations

**Smith & Krajbich 2018a:
This article investigates robustness of the core aDDM predictions across decision domains, both at the group level and the subject level. It identifies strong correlations across tasks, indicating that the effect of attention on choice is a subject-level trait. It also demonstrates that this trait is correlated with other non-choice measures of attentional scope ("tunnel vision"). Finally, it identifies cases where the aDDM does not seem to apply, i.e. decisions where subjects are simply looking for the bigger pot of money for themselves.

*Smith & Krajbich 2018b:
This article compares SSMs with either additive or multiplicative effects of attention on choice. It demonstrates that the two models exhibit a lot of mimicry, but do differ in terms of how overall value affects RTs and the effect of dwell time on choice. Using six datasets, it finds fairly consistent support for the multiplicative model, though less so in tasks with small sets of repeated stimuli.

*Pärnamets et al. 2015
In this article the authors investigate whether it is possible to bias moral judgments (e.g. "Murder is _____ justifiable. Sometimes or never?) with attention. They employ a clever design where they randomly assign a "target" option each trial, and when the target has accumulated at least 750 ms of gaze and the nontarget at least 250 ms of gaze, they prompt the subject to choose. This manipulation biases subjects' choices towards the target option.

*Holmes & Trueblood 2018
This article details advanced Bayesian methods for fitting SSMs that have changing drift rates within a trial and thus are not analytically tractable. It also discusses important issue of model "sloppiness", which is the phenomenon where multiple sets of parameters may lead to nearly identical fits to the data. This is an ongoing challenge for using these models for inference.

**Tavares et al. 2017
This article extends the aDDM to perceptual decisions involving orientation comparisons of angled lines. The first experiment documents the canonical aDDM relationships between difficulty, dwell time, last fixation, and choice. The second experiment uses the attention manipulation from Pärnamets et al. 2015 to demonstrate that these perceptual decisions can also be manipulated.

*Mullett & Stewart 2016
This article makes two important points. First, it demonstrates how the gaze-cascade effect (or late onset bias) need not imply feedback from the evidence accumulation to attention. Second, it shows that the gaze-cascade effect, together with positively skewed RTs, necessitates a SSM with a relative stopping rule (e.g. DDM) rather than an absolute stopping rule (e.g. race model).

*Ludwig & Evens 2018
This article uses a random-dot-motion task to make several important points. First, there is weak support for evidence being lost when the subject is not attending the stimulus. Second, early evidence has more effect than late evidence, further arguing against evidence leakage. Third, subjects attend more to noisier stimuli and switch away quickly when the evidence from a stimulus is stronger.

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Figures

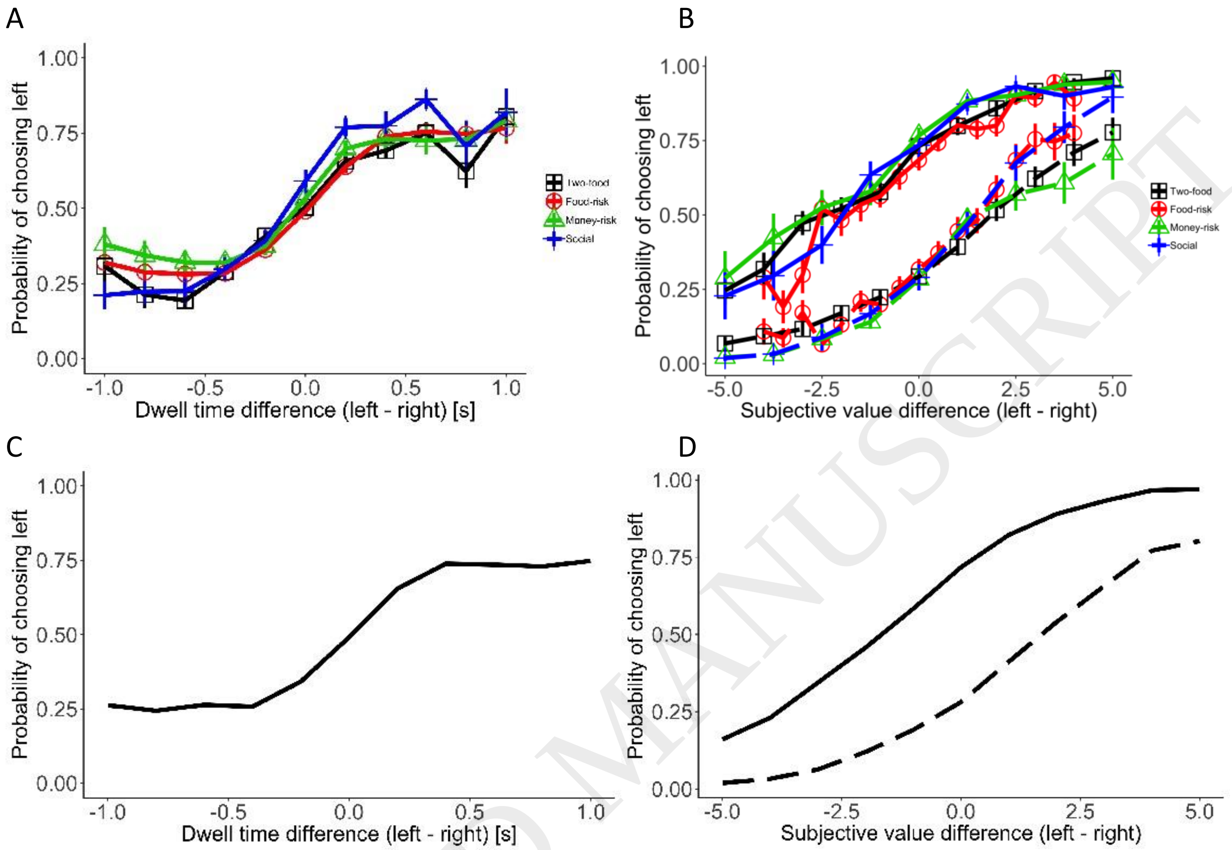

Fig. 1. Relationships between subjective value, dwell time, and choice. (a-b) Data from four choice tasks involving choice between: two foods, two 50/50 food gambles, two 50/50 monetary gambles, and two social divisions of money between oneself and a stranger [4]. **(c-d)** aDDM predictions of behavior in (a-b) using parameters from a earlier papers on two-food choice [2,37]. **(a&c)** In a given trial, the more time subjects spend looking at one option vs. the other, the more likely they are to choose that option. **(b&d)** In these plots, the solid lines indicate that left was looked at last, the dashed lines indicate that right was looked at last. For a given subjective-value difference, people are more likely to choose an option if they look at it last than if they don't. However, when an option is relatively much worse than the other, e.g. a subjective value difference of 5, then people will likely choose the better option, regardless of what they look at last.

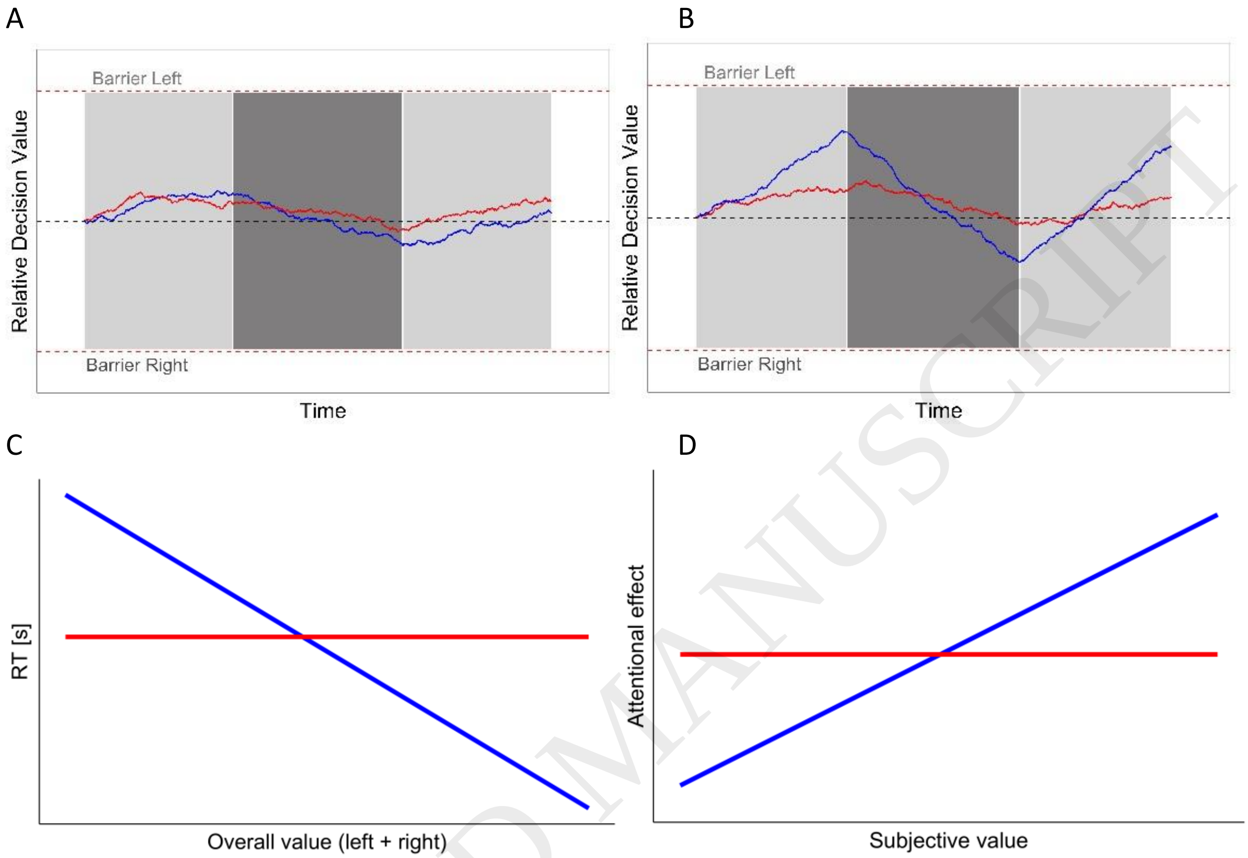

Fig. 2. Additive (red) vs. multiplicative (blue) models of attention in SSMs [15]. (a-b) Model simulations with (a) two low-value options and (b) two high-value options. Light gray regions indicate when the subject is looking left, dark gray regions when the subject is looking right. The red and blue lines indicate the evolution of the latent decision variable over time. For the additive model, the change in the drift rate (slope) due to attention does not depend on the values, i.e. it is the same between (a) and (b). For the multiplicative model, the change in drift rate due to attention is larger for higher value options. **(c-d)** Simulated model predictions for value effects on (c) RT and (d) dwell-time effects on choice. **(c)** For the additive model, RT depends only on the value difference between the two options. For a constant value difference, the overall value does not affect RT. For the multiplicative model, higher overall value leads to faster choices. **(d)** For the additive model, the effect of dwell time on choosing the attended option does not depend on the value of the option. For the multiplicative model, the effect of dwell time on choosing the attended option increases with the value of the option.